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Modeling and solving project portfolio and contractor selection problem based on project scheduling under uncertainty

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Abstract

In this paper a new formulation of the project portfolio selection problem based on the project schedules in uncertain circumstances have been proposed. The project portfolio selection models usually disregard the project scheduling, whereas is an element of the project selection process. We investigate a project portfolio selection problem based on the schedule of the projects, so that the minimum expected profit would be met in the shortest possible time period. Also due to uncertain nature of durations of the activities, this duration considered as the semi-trapezoidal fuzzy numbers. Finally, a fuzzy linear programming model is developed for the problem, where the results indicated the validity of the presented model.

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1. Introduction

The managers of the project-based organizations, confront the limited financial resources where they always face a project portfolio selection and scheduling problem. Various experimental and analytical mechanisms are developed and presented for project portfolio selection problem. Most of these tools prioritize projects based on the expert opinions about value, importance and available resources of the projects. In the different industries the common approaches and methods of the project selection mainly consists of two steps: First, all of the projects are evaluated separately and then the optimal set of the projects will be selected using a greedy algorithm. These

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projects are selected and prioritized by the series of predetermined criteria (Henriksen & Traynor, 1999; Linton, Walsh, & Morabito, 2002; Meade & Presley, 2002). Second, the projects are selected one by one according to their priorities until the resources are finished. These methods and approaches are easy and thus are widely used in practical. Nevertheless, it should be noted that the composition of the projects with higher priorities would not necessarily lead to more profitable portfolio (Chien, 2002). The major key in this model is the evaluation of the projects and the assessment of their objective value. One of the project selection policies is the selection process based on the evaluation and ranking of each project. For the ranking of the projects, there are different evaluation methods. The most widely used method is the economic analysis, in which the projects are ranked according to their present net value. On the other hand, in order to overcome the weakness of focusing on the individual criteria in the project ranking, the ranking models are proposed and used based on the several criteria to evaluate the projects (Klein, 2000). As seen the project portfolio selection models usually disregard the project scheduling, whereas is an element of the project selection process. In this paper we propose a mathematical model to consider the project portfolio selection based on the scheduling of the projects as well as the contractor selection possibility for each one of the current activities.

The rest of this paper is organized as follows. The literature is reviewed in Section 2. In Section 3, the proposed mathematical model is presented. In Section 4, the problem solving approach is proposed. As an illustration of the model, we present a numerical example in Section 5. The model results analysis is discussed in Section 6. This paper is concluded in Section 7.

2. Literature review

Many models are suggested to help the organizations to choose and schedule their projects. Dos Santos (1989) tries to represent a policy with ranking technique. The ranking method is a structured policy, which simultaneously consider several factors such as economic profit, business goals and so on. Lootsma, Mensch, & Vos (1990) and Lucas & Moore (1976) suggested the scoring method for the project selection. The scoring model can afford all of the important factors in the process of project selection and provide the theoretic indicator to select between different projects.

The mathematical models are the optimization models, which apply the mathematical programming techniques for the optimal selection of projects through the nominee projects. The selection is function of the maximization of the objective and satisfaction of the resource constraints. Badri, Davis, & Davis (2001) proposed a goal programming model for project portfolio selection in the information system projects.

Stummer & Heidenberger (2003) suggested a model and searching approach for Pareto optimal project portfolio in multi-stage decision-making process. The value assessment of the proposed portfolios are widely studied by the multidisciplinary weighting models. Gabriel, Kumar, Ordonez, & Nasserian (2006) have introduced the multi-objective optimization model with regard to the probability distribution of the costs. Abello & Michalewicz (2014) examined a special case of the resource constraint project scheduling problem, in which the number of applicable activities changes over time. In other words, unlike the common models of the resource constraint project scheduling problem, in which the number of activities is predetermined, in this case the number of activities is not fixed and varies in the progress of the project. Since the issue of the project portfolio selection and scheduling is categorized in NP-hard problems (Doerner, Gutjahr, Hartl, Strauss, & Stummer, 2004), in the recent years the meta-heuristic algorithms such as evolutionary algorithms (Medaglia, Graves, & Ringuest, 2007) and colony algorithms (Doerner, et al., 2004) are used to solve these problems. Stummer & Heidenberger (2003) applied an ant colony optimization approach to solve software project scheduling problem. In this proposed approach, with respect to the implementation of the software oriented project activities by some individuals, a mechanism is presented for the distribution of the activities and assigning them to the implementing agents.

Ghorbani & Rabbani (2009) developed a two objective model to maximize the productivity of the projects and minimize the total deviation of the allocated resources in two successive periods and for the problem solving introduced the algorithm based on the genetic algorithm, which is compared with NSGA-2 algorithm. Tasan & Gen (2013) intended to solve the project portfolio selection and scheduling problems simultaneously in separate networks, which are independently examined for the project portfolio selection and scheduling by the integrated genetic algorithm method. In this presented approach, the multi-stage decision-making approach is used. Minku, Sudholt, & Yao (2014) studied a variety of approaches for the problem of the project scheduling through the efficiency analysis and finally reached an improved approach to solve these types of the problems. This approach

is developed based on run-time analysis, in which the efficiency of the proposed approach is improved. Kazemipoor, Tavakkoli-Moghaddam, Shahnazari-Shahrezaei, & Azaron (2013) developed the differential evolution algorithm for solving the project portfolio scheduling by the multi-skilled workforce. Such problem is a developed version of the multi-objective as well as multi-mode project portfolio scheduling problem, in which the workforce have different specialties for implementing different activities. Also a new goal programming model is developed to find the minimum deviation from the average time to complete each project as well as the resources dedication. In addition to the meta-heuristic algorithms, the heuristic approaches are used for the project portfolio selection and scheduling. Messelis & De Causmaecker (2014) presented their approach for the automatic selection of the algorithm for solving the project scheduling problem with multi-mode resource constraint. The proposed approach is based on the concepts of the models, which experimentally include some difficulties. These models have the problem solving features depicted through the algorithm performance that are able to predict the performance of the algorithms. Rafiee, Kianfar, & Farhadkhani (2014) studied the multi-period project portfolio selection and scheduling problem by using the multi-stage stochastic programming approach. Artigues, Leus, & Nobibon (2013) have used the robust optimization approach for the resources constraint project scheduling problem with the uncertain duration of the activities. In this regard, a scenario relaxation algorithm and a scenario relaxation-based heuristic is developed to solve the problem.

3. Problem formulation

In this paper, we investigate a project portfolio selection problem based on the schedule of the projects, so that the minimum expected profit will be met in the shortest possible time period for the completion of the projects in the portfolio. Moreover, it is assumed that various activities in different projects are performed by different contractors, so that there is a set of authorized contractors for every activity, whereas every activity is utmost carried out by one of them. In other words, in order to determine a contractor for any activity with respect to the diversity of the contractors to carry out, different scenarios can be depicted, which by modelling represented in this paper the best scenario would be selected with respect to the authorized contractors for each activity and their scheduling, so that all of them are satisfied and the minimum expected profit is met in the shortest possible time period.

The set of indices, parameters and decision variables of the problem are as follow:

Sets and Indices:

I : The set of the projects

i, i' : Project index

J : The set of the activities

j, j' : Activity index

K : The set of the contractors

k, k' : Contractor index

J_i : Project i set of the activities

n_i : Project i last activity

k_j : The set of the contractors who can perform activity j

P_j : Activity j set of the predecessors

Parameters:

\tilde{t}_{ijk} : Duration for performing activity j of project i by contractor k considered as a semi-trapezoidal fuzzy number.

C_i : Project i profit

S : Minimum project portfolio expected profit

M : Large number

Variables:

x_{ijk} : Start time of the activity j of the project i performed by the contractor k

Z_{ijk} : Binary variable indicating that activity j of project i is performed by the contractor k or not

y_i : Binary variable indicating that project i is selected or not

y_l : Binary variable associated with l^{th} constraints

T : Latest completion time of the projects

Objective function and problem constraints:

$$\text{Min } T \quad (1)$$

$$T \geq x_{ijk} + \tilde{t}_{ijk} Z_{ijk} \quad \forall i \in I ; j = n_i ; k \in k_j \quad (2)$$

$$\sum_{k \in k_j} x_{ijk} \geq \sum_{k' \in k_j} \left(x_{i j' k'} + \tilde{t}_{i j' k'} Z_{i j' k'} \right) \quad \forall i \in I ; j, j' \in J_i ; k \in k_j ; k' \in k_{j'} ; j' \in P_j \quad (3)$$

$$x_{ijk} \leq M Z_{ijk} \quad \forall i \in I ; j \in J_i ; k \in k_j \quad (4)$$

$$x_{ijk} \geq x_{i' j' k} + \tilde{t}_{i' j' k} - M \left(2 - Z_{ijk} - Z_{i' j' k} + y_l \right) \quad \forall i, i' \in I ; j \in J_i ; j' \in J_{i'} ; j \neq j' ; k \in k_j \cap k_{j'} \quad (5)$$

$$x_{i' j' k} \geq x_{ijk} + \tilde{t}_{ijk} - M \left(2 - Z_{ijk} - Z_{i' j' k} + 1 - y_l \right) \quad \forall i, i' \in I ; j \in J_i ; j' \in J_{i'} ; j \neq j' ; k \in k_j \cap k_{j'} \quad (6)$$

$$y_l \leq \frac{1}{2} \left(Z_{ijk} + Z_{i' j' k} \right) \quad \forall i, i' \in I ; j \in J_i ; j' \in J_{i'} ; j \neq j' ; k \in k_j \cap k_{j'} \quad (7)$$

$$y_l \leq \frac{1}{2} \left(M \left(2 - Z_{ijk} - Z_{i' j' k} \right) + 2 \right) \quad \forall i, i' \in I ; i = i' ; j \in J_i ; j' \in J_{i'} ; j \neq j' ; k \in k_j \cap k_{j'} ; j \in P_{j'} \quad (8)$$

$$y_l \geq \frac{1}{2} \left(M \left(Z_{ijk} + Z_{i' j' k} - 2 \right) + 2 \right) \quad \forall i, i' \in I ; i = i' ; j \in J_i ; j' \in J_{i'} ; j \neq j' ; k \in k_j \cap k_{j'} ; j \in P_{j'} \quad (9)$$

$$\sum_{k \in k_j} Z_{ijk} \leq 1 \quad \forall i \in I ; j \in J_i \quad (10)$$

$$\sum_{i \in I} C_i y_i \geq S \quad (11)$$

$$y_i = \sum_{k \in k_j} Z_{ijk} \quad \forall i \in I ; j \in J_i \quad (12)$$

$$T, x_{ijk} \geq 0 \quad y_i, y_l, Z_{ijk} = \text{Binary} \quad (13)$$

The objective function is calculated in Eq. (1), whereas the variable T is given for the Constraint (2), which means that the project portfolio is implemented in the completion time of the last activity of the last ongoing project. The Constraint (3) represents a predecessor relationships between the project activities, which means that the start time of the successor activities should be posited after the completion time of their predecessor activities.

The Constraint (4) indicates that in the case of not allocating an activity to a particular contractor, the start time would not be set for that specific activity. When two activities are performed in one project or two different projects by the same contractor, the Constraints (5) to (9) should be satisfied, thereby it means that the both activities due to having same contractor could not overlap with each other. These constraints are activated in the case of assigning a contractor to carry out two separate activities, the start time of an activity take place after

finishing another. In condition that two activities belong to a project, the sequence of their implementation time would be based on predecessor network of the project. The Constraints (8) and (9) state such condition. The Constraint (10) states that each activity is utmost allocated to one contractor. In other words, the implementation of an activity cannot rely on two different contractors. The Constraint (11) is used to satisfy the minimum expected profit of the project portfolio. When a project is selected for inclusion in the project portfolio, all of the project activities must be performed by their authorized contractors, in other words, each activity must be carried out by a particular contractor. When a project is not selected, no contractor will be assigned to its activities. The above conditions are stated in the Constraint (12). Finally the sign constraint correspond to decision variables is mentioned in the Constraint (13).

4. Problem solving approach

We study a type of the fuzzy linear programming in which technological coefficients are semi-trapezoidal fuzzy numbers and thus the model will be as model (14):

$$\begin{aligned} & \text{Maximize } \sum_{j=1}^n c_j x_j \\ & \text{Subject to:} \\ & \sum_{j=1}^n \tilde{a}_{ij} x_j \leq b_i \quad \forall i=1,2,\dots,m; j=1,2,\dots,n \end{aligned} \quad (14)$$

Where \tilde{a}_{ij} is a fuzzy number explained as a fuzzy set in which $\mu_{\tilde{a}_{ij}}(x)$ is the membership function of \tilde{a}_{ij} as follows:

$$\tilde{a}_{ij} = \left\{ \left(x, \mu_{\tilde{a}_{ij}}(x) \right), x \in R \right\} \quad \mu_{\tilde{a}_{ij}}(x) = \begin{cases} 1 & \text{if } x \prec a_{ij} \\ \left(\frac{a_{ij} + d_{ij} - x}{d_{ij}} \right) & \text{if } a_{ij} \prec x \prec a_{ij} + d_{ij} \\ 0 & \text{if } x \succ a_{ij} + d_{ij} \end{cases}$$

For solving the fuzzy linear programming model two classical linear programming models are demanded. First without considering the tolerances of the technological coefficients and second by taking into account these tolerances where will be as following models respectively:

$$\begin{aligned} & \text{Maximize } \sum_{j=1}^n c_j x_j \\ & \text{Subject to:} \\ & \sum_{j=1}^n a_{ij} x_j \leq b_i \quad \forall i=1,2,\dots,m; j=1,2,\dots,n \end{aligned} \quad (15)$$

$$\begin{aligned} & \text{Maximize } \sum_{j=1}^n c_j x_j \\ & \text{Subject to:} \\ & \sum_{j=1}^n (a_{ij} + d_{ij}) x_j \leq b_i \quad \forall i=1,2,\dots,m; j=1,2,\dots,n \end{aligned} \quad (16)$$

The value of the objective function will be between Z_1 and Z_2 when the technological coefficients provide values between a_{ij} and $(a_{ij} + d_{ij})$. Given $Z_l = \min(Z_1, Z_2)$ and $Z_u = \max(Z_1, Z_2)$. The fuzzy objective function which is a subset of R^n will be as follows:

$$\mu_G(x) = \begin{cases} 0 & \text{if } \sum_{j=1}^n c_j x_j \prec Z_l \\ \left(\frac{\sum_{j=1}^n c_j x_j}{Z_u - Z_l} \right) & \text{if } Z_l \leq \sum_{j=1}^n c_j x_j \prec Z_u \\ 1 & \text{if } \sum_{j=1}^n c_j x_j \geq Z_u \end{cases}$$

And the i^{th} fuzzy constraint which is a subset of R^m will be as follows:

$$\mu_{C_i}(x) = \begin{cases} 0 & \text{if } b_i < \sum_{j=1}^n a_{ij} x_j \\ \left(b_i - \sum_{j=1}^n a_{ij} x_j \right) / \sum_{j=1}^n d_{ij} x_j & \text{if } \sum_{j=1}^n a_{ij} x_j \leq b_i < \sum_{j=1}^n (a_{ij} + d_{ij}) x_j \\ 1 & \text{if } b_i \geq \sum_{j=1}^n (a_{ij} + d_{ij}) x_j \end{cases}$$

For solving the problem, two approaches can be considered. First max-min method introduced by Bellman & Zadeh (1970) in which for the intersection of the fuzzy sets min operator is used. Thus:

$$\mu_D(X) = \min \left(\mu_G(X), \min_i \left(\mu_{C_i}(X) \right) \right)$$

And thus solving $\max(\mu_D(x)) = \max(\min(\mu_G(x), \min(\mu_{C_i}(x))))$ is the optimal fuzzy decision as follows:

$$\begin{aligned} & \text{Maximize } \lambda \\ & \lambda \leq \mu_G(X) \\ & \lambda \leq \mu_{C_i}(X) \quad \forall i = 1, 2, \dots, m \end{aligned}$$

In second approach product operator is used for the intersection of the fuzzy sets. Therefore in this case solving $\max(\mu_D(x)) = \max(\text{product}(\mu_G(x), (\mu_{C_i}(x))))$ will be the optimal fuzzy decision as follows:

$$\text{Maximize } \prod_{i=1}^m \mu_G(X) \times (\mu_{C_i}(X))$$

In the second approach there are less constraints which enhances the required time for solving the problem as well because of having compensation attribute the product operator has more accuracy in comparison to min operator.

5. Sample problem

In the sample problem, as shown in Figure 1, a network consists of three projects, as each one has seven activities, in which the activities of the projects can also be performed by six different contractors is considered. The network of these projects and the duration of the activities assumed as semi-trapezoidal fuzzy numbers are in the following figure.

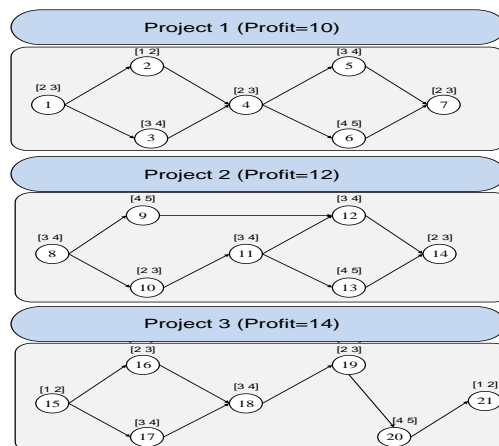


Fig. 1. The network of the projects.

Table 1 shows the predecessor relationships between activities and the list of authorized contractors for each ones.

Table 1. Predecessor relationships and Authorized Contractors of the activities.

Activity	1	2	3	4	5	6	7
Predecessor/s	-	1	1	2 3	4	4	5 6
Authorized Contractor/s	1	1	2 3	1 2	3	4	5 6
Activity	8	9	10	11	12	13	14
Predecessor/s	-	8	8	10	9 11	11	12 13
Authorized Contractor/s	5	6	3 4	2	1 6	6	3
Activity	15	16	17	18	19	20	21
Predecessor/s	-	15	15	16 17	18	19	20
Authorized Contractor/s	2 3	4 5	1 3 6	1 2 3	4	5	6

6. Problem results analysis

After the modelling and solving of the mentioned problem by Lingo Software, the scheduling of the activities and the selected contractor to perform every activity are presented in Table 2.

Table 2. Optimal solution of the problem.

Activity	Project	Start Time	Respective Contractor	Activity	Project	Start Time	Respective Contractor
1	1	1	1	8	2	1	5
2		2.5	1	9		3.5	6
3		2.5	2	10		3.5	4
4		7	1	11		7	2
5		8.5	3	12		9.5	1
6		8.5	4	13		9.5	6
7		14	5	14		14	3

According to the above figure, among the three nominee projects, Projects 1 and 2 are selected for the project portfolio, as these projects provide the least expected profit which is equal to 22 cost units. In addition, the contractor is specified to perform each activity. For example, Activity 3 is done by contractor 2. Also the scheduling of the activities are given in the above table, which represents the beginning time of each activity. The graphical diagram of the activities carried out by the contractors is shown in Figure 2.

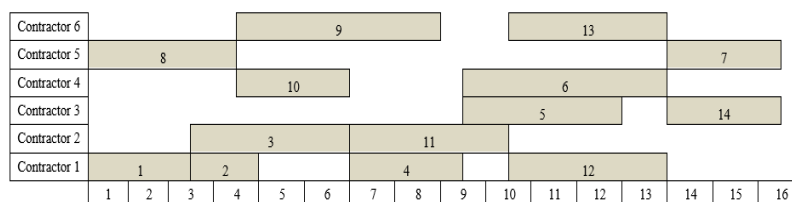


Fig. 2. The graphical diagram of the problem.

According to the above figure, non-overlapping constraint of the activities carried out by a contractor is evident. In addition, according to this figure, the scheduling of the activities is done based on the mentioned modelling, where the predecessor relationships between the activities are fulfilled. Both above cases, confirmed the validity of the model designed for the problem. Also, according to the software output and the scheduling, the minimum required time to complete the project portfolio is equal to 15.5 time units.

7. Conclusion

In this paper, the modelling and proposed solution are studied and evaluated to help the top level management

in the organization to consider the project portfolio selection process in their organization through the real impact of the projects and the related processes for the selected contractor. In the project selection process, neglecting the issue of the conflict and competition between activities of the projects, which may affect the duration of the activities and then affect the completion time of the project portfolio, leads to unrealistic scheduling. This issue points to the significance of the project portfolio selection process based on the scheduling of the each project. In other words, unlike the other techniques for the project portfolio selection, which generally use the multi-criteria decision-making approaches and the decision making criteria are disregarded in the schedule of the projects, the project portfolio selection based on the scheduling of the nominee projects and their interference effects, will be close to the real conditions and more practical. In this paper, according to the presented assumptions, a fuzzy linear programming model is presented to consider the project portfolio selection based on the scheduling of the projects as well as the contractor selection possibility for each one of the current activities. Also due to uncertain nature of durations of the activities, these durations are considered as a semi-trapezoidal fuzzy numbers. Afterward, a sample problem is designed, while the obtained results have proved the validity of the model.

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